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Electric and magnetic monopole solutions free from singularities in the non-symmetric unified field theory

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Abstract. By introducing a physically acceptable, supplementary condition for the field functions in unified field theory, the electric and magnetic monopole solutions free from singularities everywhere were deduced. This indicates that the divergence difficulties of a point particle can be removed by utilising a reasonable Boal–Moffat condition.

1. Introduction

Many attempts have been made to combine the gravitational and electromagnetic fields so that electromagnetism, as well as gravitation, appears as a property of the space–time continuum rather than as a separate physical phenomenon. One of the most important attempts is Einstein's unified field theory (Einstein 1945, Einstein and Strauss 1946) based on the non-symmetric field. Papapetrou and Schrödinger (1951), Kursunoglu (1952) and Bonnor (1951) proposed a modified non-symmetric unified field theory which led to some reasonable results. In 1975 Johnson indicated, using a fast-motion approximation, that charged particles in the Bonnor theory interact in the lowest non-trivial order of approximation through the complete laws of classical electrodynamics—Maxwell's equations for the electric and magnetic fields, the Lorentz force and the radiation force acting on the particles. Moffat and Boal (1975), Boal and Moffat (1975) and Pant (1975) have derived the exact static spherically symmetric singular solution to the field equation. Their solution represents an isolated electric particle, but they have shown that there are no magnetic monopole solutions in the non-symmetric field equations. However, I do not believe that their conclusions are valid.

In this paper we will show that both electric and magnetic monopole solutions free from singularities everywhere in the unified field theory exist, by introducing a new (Boal and Moffatt 1975) supplementary condition for the field functions.

2. Electrostatic monopole solutions free from singularities

Following the work by Moffat and Boal (1975) and Boal and Moffat (1975), the unique field equations can be written in the form

$$\partial_\alpha g_{\mu\nu} - g_{\mu\sigma}\Gamma^{\sigma}_{\alpha\nu} - g_{\sigma\nu}\Gamma^{\sigma}_{\mu\alpha} = 0, \tag{2.1}$$

$$\Gamma^{\alpha}_{[\mu\alpha]} = 0, \tag{2.2}$$

$$R^*_{(\mu\nu)}=0, \tag{2.3}$$

$$\partial_\sigma R^*_{[\mu\nu]}+\partial_\mu R^*_{[\nu\sigma]}+\partial_\nu R^*_{[\sigma\mu]}=0, \tag{2.4}$$

where

$$R^*_{\mu\nu}=R_{\mu\nu}+I_{\mu\nu}, \tag{2.5}$$

$$R_{\mu\nu}=\partial_\alpha\Gamma^\alpha_{\mu\nu}-\tfrac{1}{2}(\partial_\nu\Gamma^\alpha_{(\mu\alpha)}+\partial_\mu\Gamma^\alpha_{(\nu\alpha)})-\Gamma^\alpha_{\mu\sigma}\Gamma^\sigma_{\alpha\nu}+\Gamma^\alpha_{\mu\nu}\Gamma^\sigma_{\alpha\sigma}, \tag{2.6}$$

$$I_{\mu\nu}=-k^{-2}(g_{\mu\sigma}g^{[\sigma\rho]}g_{\nu\rho}+\tfrac{1}{2}g_{\mu\nu}g_{\sigma\rho}g^{[\sigma\rho]}+g_{[\mu\nu]}), \tag{2.7}$$

with

$$g^{\mu\nu}g_{\sigma\nu}=\delta^\mu_\sigma, \tag{2.8}$$

$$g_{[\mu\nu]}=c^{-2}k\sqrt{G}\,F_{\mu\nu}, \tag{2.9}$$

(in gaussian units)

$$g_{[\mu\nu]}=\tfrac{1}{2}(g_{\mu\nu}-g_{\nu\mu}), \tag{2.10}$$

$$g_{(\mu\nu)}=\tfrac{1}{2}(g_{\mu\nu}+g_{\nu\mu}). \tag{2.11}$$

$F_{\mu\nu}$ is the electromagnetic tensor; k, the universal constant; G, the gravitational constant and c, the velocity of light. In contrast with Moffat (1979), in our notation, $g_{[\mu\nu]}$ is a real skew-tensor, not an imaginary one.

Dividing $I_{\mu\nu}$ into symmetric and skew parts, we have

$$I_{(\mu\nu)}=-k^{-2}(g_{(\mu\rho)}g^{[\rho\sigma]}g_{[\sigma\nu]}+g_{(\nu\rho)}g^{[\rho\sigma]}g_{[\sigma\mu]}+\tfrac{1}{2}g_{(\mu\nu)}g_{[\sigma\rho]}g^{[\sigma\rho]}), \tag{2.12}$$

$$I_{[\mu\nu]}=-k^{-2}(g_{[\mu\rho]}g^{[\rho\sigma]}g_{[\sigma\nu]}+g_{(\mu\rho)}g^{[\rho\sigma]}g_{[\sigma\nu]}+\tfrac{1}{2}g_{[\mu\nu]}g_{[\rho\sigma]}g^{[\rho\sigma]}+g_{[\mu\nu]}). \tag{2.13}$$

Now, let us consider the electrostatic spherically symmetric solutions.

According to Bonnor (1951), the most general spherically symmetric tensors are given by

$$g_{\mu\nu}=\begin{bmatrix}-\alpha & 0 & 0 & w\\ 0 & -\beta & 0 & 0\\ 0 & 0 & -\beta\sin^2\theta & 0\\ -w & 0 & 0 & \gamma\end{bmatrix}, \tag{2.14}$$

$$g^{\mu\nu}=\begin{bmatrix}-\dfrac{\gamma}{\alpha\gamma-w^2} & 0 & 0 & -\dfrac{w}{\alpha\gamma-w^2}\\ 0 & -\beta^{-1} & 0 & 0\\ 0 & 0 & -(\beta\sin^2\theta)^{-1} & 0\\ \dfrac{w}{\alpha\gamma-w^2} & 0 & 0 & \dfrac{\alpha}{\alpha\gamma-w^2}\end{bmatrix} \tag{2.15}$$

$(0\leqslant r\leqslant\infty,\ 0\leqslant\theta\leqslant\pi,\ 0\leqslant\varphi\leqslant 2\pi)$.

Inserting (2.14) and (2.15) into (2.1) and (2.2) we obtain

$$\Gamma^1_{11}=\alpha'/2\alpha, \qquad \Gamma^1_{22}=\operatorname{cosec}^2\theta\,\Gamma^1_{33}=-\beta'/2\alpha,$$

$$\Gamma^1_{14}=-\Gamma^1_{41}=(w/2\alpha)[1-(\alpha\gamma/w^2)]'/(1-\alpha\gamma/w^2),$$

$$\Gamma^1_{44}=[4ww'\alpha\gamma-2w^2\alpha'\gamma-(w^2+\alpha\gamma)\alpha\gamma']/2\alpha^2(w^2-\alpha\gamma),$$

$$\Gamma^2_{12} = +\Gamma^2_{21} = \beta'/2\beta, \qquad \Gamma^2_{33} = -\sin\theta\cos\theta, \qquad \Gamma^2_{24} = -\Gamma^2_{42} = -w\beta'/2\alpha\beta,$$
$$\Gamma^3_{13} = \Gamma^3_{31} = \beta'/2\beta, \qquad \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta, \qquad \Gamma^3_{34} = -\Gamma^3_{43} = -w\beta'/2\alpha\beta, \tag{2.16}$$
$$\Gamma^4_{14} = \Gamma^4_{41} = (2ww'\alpha - w^2\alpha' - \alpha^2\gamma')/(2\alpha(w^2 - \alpha\gamma)),$$
$$[1-(\alpha\gamma/w^2)]'/[1-(\alpha\gamma/w^2)] = 2\beta'/\beta, \tag{2.17}$$

where the prime denotes differentiation with respect to the radical coordinate r.

Inserting these results into (2.3) and (2.4), we only obtain three independent equations by using the results for $R_{\mu\nu}$ by Bonnor (1951), and the results for $I_{\mu\nu}$ by Moffat and Boal (1975):

$$-(\beta'/\beta)' - \tfrac{1}{2}(\beta'/\beta) + (\alpha'\beta'/2\alpha\beta) + \Gamma^4_{14}(\alpha'/2\alpha - \Gamma^4_{14}) + (1/k^2)\alpha w^2/(\alpha\gamma - w^2) - (\Gamma^4_{14})' = 0, \tag{2.18a}$$

$$1 - (\beta'/2\alpha)' - (\beta'/4\alpha)[(w^2-\alpha\gamma)'/(w^2-\alpha\gamma)] - (1/k^2)\beta w^2/(\alpha\gamma - w^2) = 0, \tag{2.18b}$$

$$(\Gamma^1_{44})' + (w^2/4\alpha^2)[(1-\alpha\gamma/w^2)'/(1-\alpha\gamma/w^2)]^2 + w^2\beta'^2/2\alpha^2\beta^2$$
$$+ \Gamma^1_{44}(\alpha'/2\alpha - \Gamma^4_{14} + \beta'/\beta) - (1/k^2)\gamma w^2/(\alpha\gamma - w^2) = 0. \tag{2.18c}$$

The fourth equation $R_{14} + I_{14} = 0$ is not an independent equation because the contracted Bianchi identical equation $R^{*\nu}_{\mu;\nu} \equiv 0$.

In order to solve these equations, we need a supplementary condition because of the restriction of the contracted Bianchi identical equation. Following Fock's viewpoint (Fock 1955), the supplementary condition is by no means arbitrary. To obtain reasonable physical results, e.g. the definition of energy of a charged particle, we utilise the Boal–Moffat (1975) condition

$$-g_{44}g_{11} = \alpha\gamma = 1. \tag{2.19}$$

In Moffat and Boal (1975) the supplementary condition is $\beta = r^2$ which leads to a singular solution for the charged particle. However, a new supplementary condition $\alpha\gamma = 1$ is introduced in § 3 of Boal and Moffat (1975). Here the new supplementary condition $\alpha\gamma = 1$ is used.

By solving (2.18) and using the results of Moffat and Boal (1975), we find that

$$\gamma = \alpha^{-1} = (1 - 2m\beta^{-1/2} + Q^2\beta^{-1})(1 + k^2Q^2\beta^{-2}), \tag{2.20a}$$
$$\mathrm{d}\beta/\mathrm{d}r = 2\beta^{3/2}(\beta^2 + k^2Q^2)^{-1/2}, \tag{2.20b}$$
$$w = \pm kQ(\beta^2 + k^2Q^2)^{-1/2}, \tag{2.20c}$$

where $m = GM/c^2$, $Q^2 = Ge^2/c^4$ in Gaussian units.

Solving (2.20*b*), we obtain

$$r + r_0 = \frac{1}{2}\int \frac{(\beta^2 + k^2Q^2)^{1/2}}{\beta^{3/2}}\,\mathrm{d}\beta$$
$$= |kQ|^{1/2}\left[k\left(\phi, \frac{1}{\sqrt{2}}\right) - 2\varepsilon\left(\phi, \frac{1}{\sqrt{2}}\right) + \frac{2(\beta/|kQ|)^{1/2}(1+\beta^2/k^2Q^2)^{1/2}}{1 + (\beta/|kQ|)} - \left(\frac{\beta}{|kQ|}\right)^{-1/2}\left(1 + \frac{\beta^2}{k^2Q^2}\right)^{1/2}\right] \tag{2.21}$$

with

$$\cos\phi = (1-\beta/|kQ|)/(1+\beta/|kQ|),$$

$$k\left(\phi, \frac{1}{\sqrt{2}}\right) = \int_0^{\phi} \frac{d\lambda}{(1-2\sin^2\lambda)^{1/2}} \qquad \text{(first kind of elliptic integral)}, \tag{2.22}$$

$$\varepsilon\left(\phi, \frac{1}{\sqrt{2}}\right) = \int_0^{\phi} (1-\tfrac{1}{2}\sin^2\lambda)^{1/2}\, d\lambda \qquad \text{(second kind of elliptic integral)}$$

where r_0 is a constant of integration.

Making a coordinate transformation of

$$d\rho/dr = (1+k^2Q^2/\rho^4)^{-1/2} \tag{2.23}$$

we get

$$\begin{aligned} '\alpha &= (1-2m\rho^{-1}+\tfrac{1}{2}Q^2\rho^{-2})^{-1}, \\ '\gamma &= (1-2m\rho^{-1}+Q^2\rho^{-2})(1+k^2Q^2\rho^{-4}), \\ 'w &= \pm kQ\rho^{-2}. \end{aligned} \tag{2.24}$$

Taking into account $k^2Q^2 \approx 0$ and neglecting the terms k^2Q^2/r_0^5 (or $k^2Q^2/\beta^{5/3}|_{r=0}$) we get from (2.20)–(2.22)

$$\begin{aligned} &\beta = (r+r_0)^2[1+\tfrac{1}{3}k^2Q^2(r+r_0)^{-4}], \qquad \gamma = \alpha^{-1} = [1-2m\beta^{-1/2}+Q^2\beta^{-1}](1+k^2Q^2\beta^{-2}), \\ &w = \pm kQ(\beta^2+k^2Q^2)^{-1/2}, \\ &\varepsilon(r) = c^2 g_{14}(k\sqrt{G})^{-1} = \pm e(\beta^2+k^2Q^2)^{-1/2}, \\ &\lim_{\substack{\beta\to\infty \\ \gamma\to\infty}} \oint \varepsilon(r)(-\det g_{(\mu\nu)})^{1/2}\, d\theta\, d\varphi = \pm 4\pi e \qquad (0 \leqslant r \leqslant \infty). \end{aligned} \tag{2.25}$$

Under the approximation mentioned, the following equation is obtained from (2.23)

$$\rho \approx (r+r_0)[1+\tfrac{1}{3}k^2Q^2(r+r_0)^{-4}]^{1/2} \qquad (0 \leqslant r \leqslant \infty). \tag{2.26}$$

In order to determine the constant r_0, we utilise a generalising stationary 'equilibrium' condition (Ross 1972)

$$\lim_{r\to 0} [(d/dr)(g_{44}-1)] = 0 \tag{2.27}$$

and neglecting the terms of $k^2Q^2/[\beta^{5/2}]_{r\to 0}$ and the higher terms, then

$$\begin{aligned} m &= [Q^2/[\beta]_{r=0}^{1/2}] = Q^2/r_0(1+\tfrac{1}{3}k^2Q^2r_0^{-4})^{1/2} \\ &\approx Q^2/r_0[1+\tfrac{1}{6}(Gk^2Q^2/r_0^4c^4)] \end{aligned} \tag{2.28}$$

hence we have

$$Mc^2 = e^2/r_e, \tag{2.29}$$

$$\begin{aligned} &r_e = r_0[1+\tfrac{1}{6}(k^2e^2G/r_0^4c^4)], \\ &[\varepsilon(r)]_{r=0} = \pm e/\{r_0^4[1+\tfrac{1}{3}(k^2e^2G/c^4r_0^4)]^2+(kGe^2/c^4)\}^{1/2}, \end{aligned} \tag{2.30}$$

$$\begin{aligned} &[\gamma]_{r=0} = [\alpha^{-1}]_{r=0} = [1-(Ge^2/c^4r_e^2)](1+(k^2e^2G/c^4r_e^4)), \\ &\gamma = \alpha^{-1} \geqslant [1-(Ge^2/c^4r_e)](1+(k^2e^2G/c^4r_e^4)). \end{aligned} \tag{2.31}$$

From (2.31) we see that the 'event horizon' does not occur unless $Ge^2/c^4r_e^2 = 1$.

It follows from the above result that the divergence difficulties in unified field theory can be removed by utilising a new Boal–Moffat supplementary condition for the field functions. This shows that the supplementary condition is by no means arbitrary as indicated by Fock (1955).

It must be noted that the curvature tensor is also non-singular in the physical region $0 \leqslant r \leqslant \infty$. (The singularity occurs in the non-physical region $0 > r$.) In the recent work by Moffat (1979), a similar conclusion for the neutral source is derived, which shows that physical space–time, in the theory, is free of the essential singularity at $r = 0$.

3. Magnetic monopole solution free from singularities

For a static spherically symmetric magnetic monopole, we have (Bonnor 1951)

$$g_{\mu\nu} = \begin{bmatrix} -\alpha & 0 & 0 & 0 \\ 0 & -\beta & f\sin\theta & 0 \\ 0 & -f\sin\theta & -\beta\sin^2\theta & 0 \\ 0 & 0 & 0 & \gamma \end{bmatrix}, \tag{3.1}$$

$$g^{\mu\nu} = \begin{bmatrix} -\alpha^{-1} & 0 & 0 & 0 \\ 0 & -\beta/(f^2+\beta^2) & f/(f^2+\beta^2)\sin\theta & 0 \\ 0 & -f/(f^2+\beta^2)\sin\theta & -\beta/(f^2+\beta^2)\sin^2\theta & 0 \\ 0 & 0 & 0 & \gamma^{-1} \end{bmatrix}, \tag{3.2}$$

$$\begin{aligned} &\Gamma^1_{11} = \alpha'/2\alpha, \qquad \Gamma^1_{22} = \operatorname{cosec}^2\theta\,\Gamma^1_{33} = (fB - A\beta)/2\alpha, \\ &\Gamma^1_{23} = -\Gamma^1_{32} = (fA + B\beta)\sin\theta/2\alpha, \qquad \Gamma^1_{44} = \gamma'/2\alpha, \qquad \Gamma^2_{12} = \Gamma^2_{21} = A/2, \\ &\Gamma^2_{13} = -\Gamma^2_{31} = B\sin\theta/2, \qquad \Gamma^2_{33} = -\sin\theta\cos\theta, \qquad \Gamma^3_{13} = \Gamma^3_{31} = A/2, \\ &\Gamma^3_{12} = -\Gamma^3_{21} = -B\operatorname{cosec}\theta/2, \qquad \Gamma^3_{32} = \Gamma^3_{23} = \cot\theta, \qquad \Gamma^4_{41} = \Gamma^4_{14} = \gamma'/2\gamma, \end{aligned} \tag{3.3}$$

$$A = \frac{ff' + \beta\beta'}{f^2+\beta^2}, \qquad B = \frac{f\beta' - \beta f'}{f^2+\beta^2}, \tag{3.4}$$

$$\begin{aligned} R_{11} &= -A' - \tfrac{1}{2}(A^2+B^2) + A\alpha'/2\alpha + \Gamma^4_{14}((\alpha'/2\alpha) - \Gamma^4_{14}) - (\Gamma^4_{14})', \\ R_{22} &= \operatorname{cosec}^2\theta R_{33} = [(fB - \beta A)/2\alpha]' + [(fB-\beta A)(\alpha\gamma)']/4\alpha^2\gamma] \\ &\qquad + B(fA + \beta B)/2\alpha + 1, \\ R_{44} &= (\Gamma^1_{44})' + \Gamma^1_{44}(\alpha'/2\alpha - \Gamma^4_{14} + A), \\ R_{23}\operatorname{cosec}\theta &= -R_{32}\operatorname{cosec}\theta = [(fA + \beta B)/2\alpha]' \\ &\qquad - B(fB - \beta A)/2\alpha + (fA + \beta B)(\alpha' + 2\alpha\Gamma^4_{14})/4\alpha^2, \end{aligned} \tag{3.5}$$

$$\begin{aligned} &I_{11} = \alpha f^2/k^2(f^2+\beta^2), \qquad I_{44} = -\gamma f^2/k^2(f^2+\beta^2), \\ &I_{22} = \operatorname{cosec}^2\theta I_{33} = -\beta f^2/k^2(f^2+\beta^2), \\ &I_{23}\operatorname{cosec}\theta = -I_{32}\operatorname{cosec}\theta = -(f/k^2)[1 + \beta^2/(f^2+\beta^2)], \end{aligned} \tag{3.6}$$

where the prime denotes the differentiation with respect to r. Note that $k = \sqrt{2}k$ (Boal and Moffat 1975). Inserting these into the field equations and using (3.16) and (3.17) in the work by Boal and Moffat (1975), we get

$$\gamma''/2\gamma-(\alpha'\gamma'/4\alpha\gamma)-\tfrac{1}{4}(\gamma'/\gamma)^2+(A\gamma'/2\gamma)-\alpha f^2/k^2(f^2+\beta^2)=0, \tag{3.7}$$

$$-A'-\tfrac{1}{2}(A^2+B^2)+\tfrac{1}{2}A[\ln\alpha\gamma]'=0. \tag{3.8}$$

Now we substitute a magnetic monopole field of the form

$$H=\pm\mathscr{I}_m/(\alpha\gamma)^{1/2}\beta=F_{23}/(-\det g_{(\mu\nu)})^{1/2}; \tag{3.9}$$

then we have

$$g_{[23]}=f(\sin\theta)=(-\det g_{(\mu\nu)})^{1/2}Hk\sqrt{G}/c^2=\pm(kG/c^2)\mathscr{I}_m\sin\theta \tag{3.10}$$

(in gaussian units), hence

$$f=\pm k\sqrt{G}\,\mathscr{I}_m/c^2=\text{constant} \tag{3.11}$$

$$A=\beta\beta'/(f^2+\beta^2),\qquad B=f\beta'/(f^2+\beta^2). \tag{3.12}$$

It must be pointed out that there is an error in Boal and Moffat (1975). They have utilised two supplementary conditions for the field functions, i.e. $\beta=r^2$, and $\alpha\gamma=1$, which lead to the incorrect result: $\mathscr{I}_m=0$ (or $f=0$). However, in the case of spherical symmetry, only one supplementary condition is needed.

Inserting (3.11) and (3.12) into (3.7) and (3.8), we find the following equations by using the condition (2.19)

$$\gamma''+[\beta\beta'/(f^2+\beta^2)]\gamma'-[2G\mathscr{I}_m^2/c^4(f^2+\beta^2)]=0 \tag{3.13}$$

$$[\beta\beta'/(f^2+\beta^2)]'+\tfrac{1}{2}[\beta'^2/(f^2+\beta^2)]=0. \tag{3.14}$$

Solving (3.14), we have

$$\beta'=c_1(f^2+\beta^2)/\beta^{3/2}. \tag{3.15}$$

Taking into account the asymptotical condition

$$\beta\underset{r\to\infty}{\sim}r^2,\qquad \beta'\underset{r\to\infty}{\sim}2r, \tag{3.16}$$

thus we obtain

$$\beta'=2(f^2+\beta^2)/\beta^{3/2}. \tag{3.17}$$

Hence

$$\begin{aligned}2(r+r_0)&=\int\frac{\beta^{3/2}}{f^2+\beta^2}\,d\beta\\&=|f|^{1/2}\int\frac{t^{3/2}}{1+t^2}\,dt=|f|^{1/2}\left\{2t^{1/2}+\frac{1}{\sqrt{2}}\left[-\tan^{-1}\left(\frac{(2t)^{1/2}}{1-t}\right)\right.\right.\\&\left.\left.-\tanh^{-1}\left(\frac{(2t)^{1/2}}{1+t}\right)\right]\right\}\qquad (t=\beta/|f|).\end{aligned} \tag{3.18}$$

Inserting (3.17) into (3.13), we get

$$\frac{d}{d\rho}\left(\frac{d\gamma}{d\rho}\right)+\frac{2(\rho^4-2f^2)}{\rho(\rho^4+f^2)}\frac{d\gamma}{d\rho}=\frac{2G\mathscr{I}_m^2}{c^4\rho^4(1+f^2/\rho^4)^3}\qquad(\rho^2=\beta). \tag{3.19}$$

Solving this equation, we have

$$\frac{d\gamma}{d\rho}=\frac{\rho^4}{(\rho^4+f^2)^{3/2}}\left\{c_2+\frac{G\mathscr{I}_m^2}{c^4|f|^{1/2}}\left[\tfrac{1}{2}k\left(\phi,\frac{1}{\sqrt{2}}\right)-\frac{\rho/|f|^{1/2}}{(1+\rho^4/f^2)^{1/2}}\right]\right\}, \tag{3.20}$$

$$\gamma=c_3+\int\left\{\frac{\rho^4}{(\rho^4+f^2)^{3/2}}\left[c_2+\frac{\mathscr{I}_m^2G}{c^4|f|^{1/2}}\left(\tfrac{1}{2}k\left(\phi,\frac{1}{\sqrt{2}}\right)-\frac{\rho/|f|^{1/2}}{(1+\rho^4/f^2)^{1/2}}\right)\right]\right\}d\rho \tag{3.21}$$

where

$$k\left(\phi,\frac{1}{\sqrt{2}}\right)=\int_0^{\phi}\frac{d\lambda}{(1-\frac{1}{2}\sin^2\lambda)^{1/2}}, \qquad \cos\phi=(1-\rho^2/|f|)/(1+\rho^2/|f|). \tag{3.22}$$

Taking into account $k^2\approx 0$, (3.23) and (3.24) are found by assuming that $r_0^4\gg k^2G\mathscr{I}_m^2/c^4$ and neglecting the terms of $k^2G\mathscr{I}_m^2/c^4r_0^5$ (or $k^2G\mathscr{I}_m^2/c^4[\rho^5]_{r=0}$) and the higher terms:

$$k\left(\phi,\frac{1}{\sqrt{2}}\right)\approx -2|f|^{1/2}/\rho, \tag{3.23}$$

$$\gamma=\alpha^{-1}\approx c_3-c^2\rho^{-1}+G\mathscr{I}_m^2c^{-4}\rho^{-2}. \tag{3.24}$$

Making the asymptotic condition

$$\gamma \underset{r\to\infty}{\sim} 1-2GMc^{-2}r^{-1}+G\mathscr{I}_m^2c^{-4}r^{-2}, \tag{3.25}$$

we obtain

$$c_2=2GM/c^2, \qquad c_3=1. \tag{3.26}$$

Hence we have

$$\begin{aligned}\gamma&=\alpha^{-1}\approx 1-2GM/c^2\rho+G\mathscr{I}_m^2/c^4\rho^2,\\ \rho&\approx(r+r_0)\{1-\tfrac{1}{3}[k^2G\mathscr{I}_m^2/c^4(r+r_0)^4]\},\end{aligned} \tag{3.27}$$

$$f=\pm k\mathscr{I}_m\sqrt{G}/c^2, \tag{3.28}$$

$$\begin{aligned}&H=\pm\mathscr{I}_m/\rho^2,\\ &\oint H(-\det g_{(\mu\nu)})^{1/2}\,d\theta\,d\varphi=\pm4\pi\mathscr{I}_m \qquad (0\leqslant r\leqslant\infty).\end{aligned} \tag{3.29}$$

By using the stationary 'equilibrium' condition (2.27), we have

$$\begin{aligned}Mc^2&=\mathscr{I}_m^2/(\rho)_{r=0}\\ &=\mathscr{I}_m^2/r_0[1-\tfrac{1}{3}(k)^2G\mathscr{I}_m^2/c^4r_0^4]\\ &\equiv\mathscr{I}_m^2/r_m;\end{aligned} \tag{3.30}$$

thus

$$[\alpha^{-1}]_{r=0}=[\gamma]_{r=0}=1-G\mathscr{I}_m^2/c^4r_m^2, \tag{3.31}$$

$$[H]_{r=0}=\pm\mathscr{I}_m/r_m^2. \tag{3.32}$$

From the results obtained above we see that there exists a magnetic monopole solution free from singularities everywhere. This indicates that the divergence difficulties in the unified field theory can be removed by utilising a new (Boal and Moffatt 1975) supplementary condition for the field functions.

It must be pointed out that, in our opinion, the Moffat and Boal motion equations (see equation (4.19) of Johnson 1975) actually describe the motion of the magnetic monopoles because the magnetic solutions of $g_{[st]} = k\varepsilon_{stn}\phi_{,n}$, $\phi = \Sigma\, r^{-1}$ constant and $g_{[54]} = 0$ are chosen in their work.

4. Conclusion

In conclusion, utilising a new Boal–Moffat supplementary condition, we have obtained the following results

(a) There exists an electrostatic solution free from singularities everywhere describing an electrostatic monopole with definite energy.

(b) There exists a magnetostatic solution free from singularities everywhere describing a magnetostatic monopole with definite energy.

(c) From the results which we have obtained above we see that the divergence difficulties in the unified field theory can be removed by introducing a physically reasonable Boal–Moffat supplementary condition for the field functions.

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